Low scale inflation and the curvaton mechanism

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The primordial curvature perturbation may be due to a 'curvaton' field, which dominates (or almost dominates) the energy density before it decays. In the simplest version of the curvaton model the scale of inflation has to be quite high corresponding to a Hubble parameter $H > 10^7$ GeV. We here explore two modifications of the curvaton model which can instead allow inflation at a low scale. (i) The curvaton is a Pseudo Nambu-Goldstone Boson (PNGB), with a symmetry-breaking phase transition during inflation. (ii) The curvaton mass increases suddenly at some moment after the end of inflation but before the onset of the curvaton oscillations. Both proposals can work but not in a completely natural way. Also, the lower bound on the scale of inflation depends somewhat on the details of the framework used. Nevertheless, we show that inflation with H as low as 1 TeV or lower is possible to attain.

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I. INTRODUCTION

The primordial curvature perturbation is generated presumably from the perturbation of some scalar field, which in turn is generated from the vacuum fluctuation during inflation. The scalar field responsible for the primordial curvature perturbation is traditionally supposed to be the inflaton field, i.e. the field responsible for the dynamics and, in particular, the end of inflation [1]. This 'inflaton hypothesis' is economical, but it is quite difficult to implement and, if many scalar fields exist, it presumably is not particularly likely. An alternative is that the curvature perturbation is generated by a 'curvaton' field, which dominates (or almost dominates) the energy density before it decays [2, 3] (see also [4]). According to this 'curvaton hypothesis', the contribution of the inflaton to the curvature perturbation is negligible. This is especially true if the energy scale of inflation is much lower than the scale of grand unification, which is the typical requirement of the traditional inflaton hypothesis[1]. In fact, one of the advantages of the curvaton scenario is the relaxation of the constraints on the inflationary energy scale, which alleviates many tuning problems in inflation model–building and allows for the construction of realistic, theoretically well-motivated inflation models [6].

In the simplest version of the curvaton model though, the scale of inflation is still required to be quite high corresponding to Hubble parameter $H > 10^7$ GeV [7]. The purpose of this paper is to systematically explore some modifications of the curvaton model which can instead allow inflation at an even lower scale. To be specific, we aim for $H \sim 10^3$ GeV, which holds if the inflationary potential is generated by some mechanism of gravity-mediated supersymmetry breaking which holds also in the vacuum.

We begin by presenting some known bounds in a unified notation. Then we consider two possibilities: (i) that the curvaton is a Pseudo Nambu-Goldstone Boson (PNGB), whose order parameter is increased after the cosmological scales exit the horizon during inflation but before the onset of the curvaton oscillations and (ii) that the curvaton mass increases suddenly at some moment after the end of inflation but before the onset of the curvaton oscillations. We conclude with a prognosis for the viability of low-scale inflation within the curvaton model.

Throughout our paper we use units such that $\hbar = c = k_B = 1$ and Newton's gravitational constant is $8\pi G = m_P^{-2}$, with $m_P = 2.4 \times 10^{18}$ GeV being the reduced Planck mass.

II. THE BOUNDS ON THE SCALE OF INFLATION

In this section we present four bounds on the scale of inflation, in terms of three parameters which encode possible modifications of the simplest curvaton scenario. These bounds have been presented at least implicitly in earlier works [7, 8, 9] but not in the unified notation that we employ. The advantage of this notation is that it will allow us to compare the bounds in various situations, establishing with ease which is the most crucial. The three parameters are

- The ratio $\varepsilon \equiv \sigma_*/\sigma_{\rm osc}$, where σ_* is the value of the curvaton field at horizon exit and $\sigma_{\rm osc}$ is its value when it starts to oscillate.
- The ratio $f \equiv H_{\rm osc}/\tilde{m}_\sigma$, where $H_{\rm osc}$ is the Hubble parameter at the start of the oscillation and $\tilde{m}_\sigma$ is the effective curvaton mass after the onset of the oscillation.

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[1] Although some exceptions exist, e.g. see Ref. [5].

- The ratio $\delta \equiv \sqrt{H_{\rm osc}/H_*}$ where H_* is the Hubble parameter during inflation.

A. Curvaton physics considerations

The observed value of the nearly scale-invariant spectrum of curvature perturbations is $\mathcal{P}_\zeta = (5\times 10^{-5})^2$, which we denote simply by ζ^2. In the curvaton scenario ζ is given by [2, 10]

$$\zeta \sim \Omega_{\rm dec}\zeta_\sigma \; , \tag{1}$$

where $\Omega_{\rm dec} \leq 1$ is the density fraction of the curvaton density ρ_σ over the total density of the Universe ρ at the time of the decay of the curvaton:

$$\Omega_{\rm dec} \equiv \left.\frac{\rho_\sigma}{\rho}\right|_{\rm dec} , \tag{2}$$

and ζ_σ is the curvature perturbation of the curvaton field σ, which is [11]

$$\zeta_\sigma \sim \left.\frac{\delta\sigma}{\sigma}\right|_{\rm dec} \sim \left.\frac{\delta\sigma}{\sigma}\right|_{\rm osc} , \tag{3}$$

where 'osc' denotes the time when the curvaton oscillations begin and 'dec' denotes the time of curvaton decay.

In all the cases, which we consider,

$$\left.\frac{\delta\sigma}{\sigma}\right|_* \simeq \left.\frac{\delta\sigma}{\sigma}\right|_{\rm osc} , \tag{4}$$

where '*' denotes the epoch when the cosmological scales exit the horizon during inflation. The above typically holds true because the curvaton (being a light field) is frozen during and after inflation until the onset of its oscillations. However, this does not mean that $\sigma_* \simeq \sigma_{\rm osc}$ necessarily. Indeed, in the case of the PNGB curvaton (case (i)), with a varying order parameter v, the curvaton field is associated with the angular displacement θ from the minimum of its potential as

$$\sigma \equiv \sqrt{2}\, v\theta \; . \tag{5}$$

Therefore, even though after the end of inflation, θ remains approximately frozen (the angular motion is over damped), we may have $\varepsilon \ll 1$, where

$$\varepsilon \equiv \frac{\sigma_*}{\sigma_{\rm osc}} , \tag{6}$$

because [cf. Eq. (5)] $v_* = \varepsilon v_{\rm osc} \ll v_{\rm osc}$. However, in this case too, for the curvaton fractional perturbation we find

$$\left.\frac{\delta\sigma}{\sigma}\right|_* = \left.\frac{\delta\theta}{\theta}\right|_* \simeq \left.\frac{\delta\sigma}{\sigma}\right|_{\rm osc} , \tag{7}$$

which agrees nicely with Eq. (4).

Now, for the perturbation of the curvaton we have

$$\delta\sigma_* = \frac{H_*}{2\pi} , \tag{8}$$

Combining Eqs. (6) and (8) we find

$$\delta\sigma_{\rm osc} \simeq \frac{H_*}{2\pi\varepsilon} , \tag{9}$$

which means that, if the order parameter of a PNGB curvaton grows, the curvaton perturbation is amplified by a factor ε^{-1}.

From Eqs. (1) and (3) we have

$$\sigma_{\rm osc} \sim (\Omega_{\rm dec}/\zeta)\,\delta\sigma_{\rm osc} \; . \tag{10}$$

Using Eq. (9), we can recast the above as

$$\sigma_{\rm osc} \sim \frac{H_*\Omega_{\rm dec}}{\pi\varepsilon\zeta} . \tag{11}$$

B. The main bound on the scale of inflation

For the density fraction at the onset of the curvaton oscillations we have:

$$\left.\frac{\rho_\sigma}{\rho}\right|_{\rm osc} \sim f^{-2}\left(\frac{\sigma_{\rm osc}}{m_P}\right)^2 , \tag{12}$$

where

$$f \equiv \frac{H_{\rm osc}}{\tilde{m}_\sigma} , \tag{13}$$

and we used that $(\rho_\sigma)_{\rm osc} \simeq \frac{1}{2}\tilde{m}_\sigma^2\sigma_{\rm osc}^2$ and $\rho_{\rm osc} = 3H_{\rm osc}^2 m_P^2$. Here, $\tilde{m}_\sigma$ denotes the effective mass of the curvaton *after* the onset of its oscillations. In the basic setup of the curvaton hypothesis this effective mass is the bare mass m_σ. If this is the case then $\tilde{m}_\sigma = m_\sigma \simeq H_{\rm osc}$ (i.e. $f \simeq 1$). However, in the heavy curvaton scenario (case (ii)), the mass of the curvaton is supposed to be suddenly incremented at some time after the end of the inflationary epoch due to a coupling of the form $\lambda\chi^2\sigma^2$ with a field χ which acquires a large vacuum expectation value (VEV) at some time after the end of inflation. In this case $\tilde{m}_\sigma^2 = m_\sigma^2 + \lambda\langle\chi\rangle^2 \approx \lambda\langle\chi\rangle^2 \gg H_{\rm osc}^2$ (i.e. $f \ll 1$).

Now, we need to consider separately the cases when the curvaton decays before it dominates the Universe ($\Omega_{\rm dec} \ll 1$) or after it does so ($\Omega_{\rm dec} \sim 1$). Note, that the WMAP constraints on non-gaussianity in the Cosmic Microwave Background Radiation (CMBR) impose a lower bound on $\Omega_{\rm dec}$, which allows the range [10, 12]:

$$0.01 \leq \Omega_{\rm dec} \leq 1. \tag{14}$$

Because of the above bound we require that the density ratio ρ_σ/ρ grows substantially after the end of inflation. Typically, in the curvaton scenario this does indeed take

place after the curvaton begins oscillating, but only if the curvaton oscillates in a quadratic potential during the radiation era. As it was shown in Ref. [11], if the curvaton oscillates in a quartic or even higher order potential, its density ratio does not increase with time (it may well decrease instead) and satisfying the bound in Eq. (14) is very hard. Due to this fact, in the following, unless stated otherwise, we assume that at least a part of the period of oscillations occurs in the radiation era with a quadratic potential. Hence, we consider that $H_{\rm dec} \leq \Gamma_{\rm inf}$. We also consider the minimal scenario that the Universe, after the end of inflation, undergoes a period of matter domination (due to coherent inflaton oscillations) until reheating, when it becomes radiation dominated.

Suppose, at first, that the curvaton decays before dominating the density of the Universe so that $\Omega_{\rm dec} \ll 1$. Assuming that the curvaton oscillates in a quadratic potential, during the radiation epoch, its density fraction grows as $\rho_\sigma/\rho \propto H^{-1/2}$. Therefore, at curvaton decay we have

$$\Omega_{\rm dec} \sim \min\left\{1, \sqrt{\Gamma_{\rm inf}/H_{\rm osc}}\right\} \frac{\tilde{m}_\sigma^2 \sigma_{\rm osc}^2}{T_{\rm dec} H_{\rm osc}^{3/2} m_P^{3/2}}\,, \qquad (15)$$

where we used Eq. (12) and also that $\rho_{\rm dec} \sim T_{\rm dec}^4$, with $\Gamma_{\rm inf}$ being the decay rate of the inflaton field, which determines the reheat temperature as $T_{\rm reh} \sim \sqrt{\Gamma_{\rm inf}\, m_P}$. Using Eq. (11) the above can be recast as

$$H_* \sim \pi\varepsilon\zeta f \frac{m_P}{\sqrt{\Omega_{\rm dec}}} \left(\frac{H_{\rm dec}}{H_{\rm osc}}\right)^{1/4} \max\left\{1, \frac{H_{\rm osc}}{\Gamma_{\rm inf}}\right\}^{1/4}, \qquad (16)$$

where we used that $T_{\rm dec}^2 \sim H_{\rm dec} m_P$.

Now, suppose that the curvaton decays after it dominates the Universe so that $\Omega_{\rm dec} \sim 1$. Since $(\rho_\sigma/\rho)_{\rm dom} \simeq 1$ by definition, using again that, during the radiation epoch, $\rho_\sigma/\rho \propto H^{-1/2}$ and in view of Eq. (12), we obtain

$$H_{\rm dom} \sim \min\{H_{\rm osc}, \Gamma_{\rm inf}\} f^{-4} \left(\frac{\sigma_{\rm osc}}{m_P}\right)^4, \qquad (17)$$

where 'dom' denotes the time of curvaton domination. Employing again Eq. (11), the above can be written as

$$H_* \sim \pi\varepsilon\zeta f m_P \left(\frac{H_{\rm dom}}{H_{\rm osc}}\right)^{1/4} \max\left\{1, \frac{H_{\rm osc}}{\Gamma_{\rm inf}}\right\}^{1/4}. \qquad (18)$$

Combining Eqs. (16) and (18) we find that, in all cases

$$H_* \sim \pi\varepsilon\zeta f \frac{m_P}{\sqrt{\Omega_{\rm dec}}} \left(\frac{\max\{H_{\rm dom}, H_{\rm dec}\}}{H_{\rm osc}}\right)^{1/4} \times \max\left\{1, \frac{H_{\rm osc}}{\Gamma_{\rm inf}}\right\}^{1/4}. \qquad (19)$$

This can be rewritten as

$$H_* \sim \Omega_{\rm dec}^{-2/5} \left(\frac{H_*}{\min\{H_{\rm osc}, \Gamma_{\rm inf}\}}\right)^{1/5} \times \left(\frac{\max\{H_{\rm dom}, H_{\rm dec}\}}{H_{\rm BBN}}\right)^{1/5} (\pi\varepsilon\zeta f)^{4/5} (T_{\rm BBN}^2 m_P^3)^{1/5}, \qquad (20)$$

or equivalently (using $V_*^{1/4} \sim \sqrt{H_* m_P}$)

$$V_*^{1/4} \sim \Omega_{\rm dec}^{-1/5} \left(\frac{H_*}{\min\{H_{\rm osc}, \Gamma_{\rm inf}\}}\right)^{1/10} \times \left(\frac{\max\{H_{\rm dom}, H_{\rm dec}\}}{H_{\rm BBN}}\right)^{1/10} (\pi\varepsilon\zeta f)^{2/5} (T_{\rm BBN} m_P^4)^{1/5}, \qquad (21)$$

where 'BBN' denotes the epoch of Big Bang Nucleosynthesis (BBN) ($T_{\rm BBN} \sim 1$ MeV). Now, according to Eq. (14) we have $\Omega_{\rm dec} \leq 1$. Also, we require that the curvaton decays before BBN, i.e. $H_{\rm dec} > H_{\rm BBN}$. Moreover, we also have $\Gamma_{\rm inf} \leq H_*$. Hence, the above provides the following bounds

$$\boxed{\begin{aligned} H_* &> (\pi\varepsilon\zeta f)^{4/5} (T_{\rm BBN}^2 m_P^3)^{1/5} \sim (\varepsilon f)^{4/5} \times 10^7 \text{ GeV}, \\ V_*^{1/4} &> (\pi\varepsilon\zeta f)^{2/5} (T_{\rm BBN} m_P^4)^{1/5} \sim (\varepsilon f)^{2/5} \times 10^{12} \text{ GeV}. \end{aligned}} \qquad (22)$$

In the standard setup of the curvaton scenario $\varepsilon = f = 1$ and the above bounds do not allow inflation at low energy scales to take place [7]. However, we see that if either ε or f are much smaller than unity the lower bound on the inflationary scale can be substantially relaxed and low scale inflation can be accommodated. Still, though, there are more bounds to be considered.

C. Other bounds related to curvaton decay

Firstly, let us consider the bound coming from the fact that the decay rate of the curvaton field cannot be arbitrarily small. Indeed, in view of the fact that the curvaton interactions are at least of gravitational strength, we find the following decay rate for the curvaton

$$\Gamma_\sigma \approx \gamma_\sigma \frac{\tilde{m}_\sigma^3}{m_P^2} \leq \tilde{m}_\sigma, \qquad (23)$$

where $\gamma_\sigma \gtrsim 1$.

Suppose, at first, that the curvaton decays after the onset of its oscillations, as in the basic setup of the curvaton scenario. In this case, $\Gamma_\sigma \leq H_{\rm osc}$ and $H_{\rm dec} = \Gamma_\sigma$. Using the fact that $\max\{H_{\rm dom}, \Gamma_\sigma\} \geq \Gamma_\sigma$, Eq. (23) suggests

$$\frac{\max\{H_{\rm dom}, H_{\rm dec}\}}{H_{\rm osc}} \geq \gamma_\sigma f^{-1} \left(\frac{\tilde{m}_\sigma}{m_P}\right)^2. \qquad (24)$$

Including the above into Eq. (19) the latter becomes

$$H_* \geq \sqrt{\gamma_\sigma} (\pi\varepsilon\zeta)^2 \sqrt{f} \frac{m_P}{\Omega_{\rm dec}} \left(\frac{H_{\rm osc}}{H_*}\right) \max\left\{1, \frac{H_{\rm osc}}{\Gamma_{\rm inf}}\right\}^{1/2}, \qquad (25)$$

which results in the bounds

$$\boxed{\begin{aligned} H_* &\geq (\pi\varepsilon\zeta)^2\sqrt{f}\;\delta^2\,m_P \sim \varepsilon^2\sqrt{f}\;\delta^2\times 10^{11}\text{ GeV},\\ V_*^{1/4} &\geq \pi\varepsilon\zeta f^{1/4}\delta\,m_P \sim \varepsilon f^{1/4}\delta\times 10^{14}\text{ GeV}\;, \end{aligned}} \tag{26}$$

where we have defined

$$\delta \equiv \sqrt{\frac{H_{\rm osc}}{H_*}} \leq 1\;. \tag{27}$$

In the case of a PNGB curvaton (case (i)) we see that the bounds in Eq. (26) are drastically reduced with ε, compared with the bounds in Eq. (22). This is not so in the heavy curvaton case (case (ii)), where it is also possible that δ is not very small.

Now, provided we demand that the curvaton field does not itself result in a period of inflation, we see that the curvaton cannot dominate the Universe before the onset of its oscillations. This results into the constraint

$$\left.\frac{\rho_\sigma}{\rho}\right|_{\rm osc} \leq 1 \;\Leftrightarrow\; \tilde{m}_\sigma \leq \pi\varepsilon\zeta\,\delta^2\frac{m_P}{\Omega_{\rm dec}} \;\Leftrightarrow\; f \geq \frac{\Omega_{\rm dec}H_*}{(\pi\varepsilon\zeta)m_P}, \tag{28}$$

where we used Eqs. (11), (12), (13) and (27). Inserting the above into Eq. (25) we obtain

$$H_* \geq \gamma_\sigma(\pi\varepsilon\zeta)^3\delta^4\,\frac{m_P}{\Omega_{\rm dec}}\max\left\{1, H_{\rm osc}/\Gamma_{\rm inf}\right\}, \tag{29}$$

which results in the bounds

$$\boxed{\begin{aligned} H_* &\geq (\pi\varepsilon\zeta)^3\delta^4 m_P \sim \varepsilon^3\delta^4\times 10^{7}\text{ GeV},\\ V_*^{1/4} &\geq (\pi\varepsilon\zeta)^{3/2}\delta^2 m_P \sim \varepsilon^{3/2}\delta^2\times 10^{12}\text{ GeV}\;. \end{aligned}} \tag{30}$$

A similar bound is reached with the use of the upper bound on $\tilde{m}_\sigma$

$$\tilde{m}_\sigma \leq \gamma_\sigma^{-1/3}(H_{\rm osc}m_P^2)^{1/3}, \tag{31}$$

which comes from $\Gamma_\sigma \leq H_{\rm osc}$ and the Eq. (23), instead of the bound in Eq. (28). Inserting the above into Eq. (25) one finds [cf. Eq. (29)]

$$H_* \geq \gamma_\sigma(\pi\varepsilon\zeta)^3\delta^4\,\frac{m_P}{\Omega_{\rm dec}^{3/2}}\max\left\{1, H_{\rm osc}/\Gamma_{\rm inf}\right\}^{3/4}, \tag{32}$$

which, again, results into the bound in Eq. (30), as it was suggested in Ref. [9].

In the heavy curvaton scenario (case (ii)) we have $\varepsilon = 1$ and also $H_{\rm osc} \simeq \min\{H_{\rm pt}, \tilde{m}_\sigma\}$, where $H_{\rm pt}$ corresponds to the phase transition which increases the effective mass of the curvaton. Then, if $\delta \to 1$, the bounds in Eq. (30) are not possible to be relaxed below the standard case discussed in Ref. [7] despite the fact that we may have $f \ll 1$ in Eq. (22). Therefore, in the heavy curvaton scenario we require $\delta \ll 1$, i.e. *the onset of the curvaton oscillations has to occur much later than the end of inflation* so that $H_* \gg H_{\rm osc} \geq \Gamma_\sigma$. In this case, as can be seen in Eq. (30), it is easy to lower the bound on the inflationary scale even for a not-so-small value of δ. This is a very nice feature of this scenario. Note also, that in the case of a PNGB curvaton (case (i)) $H_{\rm osc} \sim m_\sigma \ll H_*$ and δ is very small necessarily. Because, in this case, $\varepsilon \ll 1$, it is straightforward to see that the bounds in Eq. (30) are much weaker than the bounds in Eq. (22).

As it was pointed out in Ref. [9], the sudden increment in the curvaton mass might lead to a growth in the curvaton decay rate enough for $\Gamma_\sigma > H_{\rm pt}$. This would force the curvaton to decay immediately and we can write $H_{\rm osc} \sim H_{\rm pt} \sim H_{\rm dec}$. Obviously, in this case we cannot have $H_{\rm dec} < H_{\rm dom}$ and there is no period when $\rho_\sigma/\rho \propto H^{-1/2}$. This means that $(\rho_\sigma/\rho)_{\rm osc} \sim \Omega_{\rm dec}$. Using Eqs. (11) and (12) it is easy to find

$$H_* \sim \pi\varepsilon\zeta f\frac{m_P}{\sqrt{\Omega_{\rm dec}}}, \tag{33}$$

which results in the following bounds

$$\boxed{\begin{aligned} H_* &\geq \pi\varepsilon\zeta f\,m_P \sim \varepsilon f\times 10^{14}\text{ GeV},\\ V_*^{1/4} &\geq \sqrt{\pi\varepsilon\zeta f}\,m_P \sim (\varepsilon f)^{1/2}\times 10^{16}\text{ GeV}. \end{aligned}} \tag{34}$$

It is evident that the above bounds may challenge the COBE constraint for the curvaton scenario [6] and may lead to excessive curvature perturbations from the inflaton field if ε and/or f are not much smaller than unity.

Note, that the bounds in Eq. (34) are, in general, valid in the case when $H_{\rm dec} > \Gamma_{\rm inf}$ because, in this case $(\rho_\sigma/\rho)_{\rm osc} \sim \Omega_{\rm dec}$. Indeed, combining Eqs. (19) and (33) we obtain the generic condition

$$\begin{aligned} H_* \;\sim\; & \pi\varepsilon\zeta f\frac{m_P}{\sqrt{\Omega_{\rm dec}}}\left(\frac{\max\{H_{\rm dec}, H_{\rm dom}\}}{H_{\rm osc}}\right)^{1/4}\times \\ & \times\;\min\left\{1, \frac{\Gamma_{\rm inf}}{H_{\rm dec}}\right\}^{1/4}\max\left\{1, \frac{H_{\rm osc}}{\Gamma_{\rm inf}}\right\}^{1/4}. \end{aligned} \tag{35}$$

The bounds in Eqs. (22), (26), and (33) provide the basis for our investigation, leaving the fourth bound in Eq. (34) to be considered in a companion paper [13]. As a matter of completeness we have considered all the other possible bounds coming from the requirements that $\Gamma_\sigma < \tilde{m}_\sigma$ and $H_{\rm dec} \geq H_{\rm BBN}$. We have found that these bounds lead to consistent and/or weaker constraints than the above four.

III. THE CASE OF A PNGB CURVATON WITH A VARYING ORDER PARAMETER

We discuss here the case of a pseudo Nambu-Goldstone boson (PNGB) as curvaton. Such a curvaton has the

additional advantage of being protected by the (approximate) global U(1) symmetry, which means that its flatness can be protected by the effect of supergravity corrections. Candidates for such a PNGB curvaton have been discussed in Ref. [14]. However, in contrast to those models, we consider a PNGB curvaton, whose radial field has a larger expectation value in the vacuum than at the time when the cosmological scales exit the horizon during inflation. Thus, the potential for the curvaton field σ is

$$V(\sigma) = (v\tilde{m}_\sigma)^2\Big[1-\cos\Big(\frac{\sigma}{v}\Big)\Big] \Rightarrow V(|\sigma|<v) \simeq \tfrac{1}{2}\tilde{m}_\sigma^2\sigma^2, \quad (36)$$

where $v = v(t)$ is the expectation value of the radial field and $\tilde{m}_\sigma = \tilde{m}_\sigma(v)$ is the mass of the curvaton at a given moment. In the true vacuum we have $v = v_0$ and $\tilde{m}_\sigma = m_\sigma$ with v_0 being the VEV of the radial field and m_σ being the mass of the curvaton in the vacuum.

To simplify our study *we assume that the curvaton mass has already assumed its vacuum value before the onset of the curvaton oscillations*. This means that at the onset of the curvaton oscillations $v \to v_0$ and the mass of the curvaton has its vacuum value m_σ. We further assume that $m_\sigma \leq H_*$ so that the oscillations begin after the end of inflation (were it otherwise the curvaton density would be exponentially diluted). Therefore, the curvaton oscillations begin when $H_{\rm osc} \sim m_\sigma$, which means that $f \sim 1$ and $\delta \ll 1$ [cf. Eqs. (13) and (27) respectively]. Consequently, in view of Eqs. (22), (26) and (30), we find that the most stringent lower bound on the scale inflation is given by Eq. (22) [2].

Since $f \sim 1$, the only way one can obtain low–scale inflation is a very small value of ε. However, ε cannot be arbitrarily small. In fact, we may obtain a lower bound on ε as follows:

$$\frac{\delta\sigma_*}{\sigma_*} \leq 1 \quad \Rightarrow \quad \varepsilon \geq \varepsilon_{\rm MIN} \equiv \frac{H_*}{v_0}\,, \quad (37)$$

where we have used Eqs. (5), (6) and (8) and that $\sigma_{\rm osc} \lesssim v_0$.

Another bound on ε can be obtained with the use of Eq. (28), which can be recast as:

$$\varepsilon \geq \varepsilon'_{\rm MIN} \equiv \frac{\Omega_{\rm dec}}{\pi\zeta\delta^2}\frac{m_\sigma}{m_P}\,. \quad (38)$$

Comparing the two bounds we find that $\varepsilon_{\rm MIN} > \varepsilon'_{\rm MIN}$ can be ensured if

$$v_0 < (\pi\zeta)\,\Omega_{\rm dec}^{-1} m_P\,, \quad (39)$$

which is always satisfied for $v_0 \leq 10^{14}$ GeV. For larger values of v_0 the bounds are comparable but only if m_σ is not much smaller than H_*. Indeed, it is easy to see that, if $m_\sigma < 10^{-4}H_*$ then the bound in Eq. (37) is the tightest for all $v_0 \leq m_P$. Therefore, in the following we will consider $\varepsilon_{\rm MIN}$ as the appropriate lower bound.

From Eqs. (22) and (37) and after a little algebra it is easy to get

$$V_*^{1/4} \geq \left(\frac{m_P}{v_0}\right)^2 10^{-13}{\rm GeV} \Rightarrow H_* \geq \left(\frac{m_P}{v_0}\right)^4 10^{-44}{\rm GeV}, \quad (40)$$

which means that, in principle, *the larger v_0 is the smaller $V_*^{1/4}$ can become*. For example, if $v_0 \sim 10^{10}$ GeV, then $V_*^{1/4}$ can become as low as TeV. In practice, however, such low values for the lower bound are difficult to attain.

A. Constraining the evolution of the radial field

We now turn our attention to the evolution of the radial field, which determines the value $v(t)$. We will denote the radial field with ϕ. The value of ϕ during inflation determines v_*, while its VEV is v_0.

At first it may seem that all that is required is that the radial field ϕ has a much smaller value v_*, when the cosmological scales exit the horizon during inflation, than its value v_0 at the onset of the oscillations of the curvaton. How ϕ changes from v_* to v_0 seems not to be important. However, it turns out that there is indeed an important constraint on the behaviour of the radial field, coming from the requirements of the spectral index of density perturbations[3].

Roughly, the reason is the following. The amplitude of the density perturbations is determined by the magnitude of the perturbations of the curvaton field, which, in this scenario, apart from the scale of H_* is also determined by the amplification factor ε^{-1}. The latter is determined by the value of the radial field when the curvaton quantum fluctuations exit the horizon during inflation. A strong variation of the value of the radial field results in a strong dependence of $\varepsilon(k)$ on the comoving momentum scale k, which would reflect itself on the perturbation spectrum threatening significant departure from scale invariance.

Let us quantify this. By definition the spectral index of the density perturbations is

$$n_s - 1 = \frac{d\ln(\delta\rho/\rho)}{d\ln k}. \quad (41)$$

Now, since $\delta\rho/\rho \approx \frac{2}{5}\zeta$ and $\zeta \propto \zeta_\sigma$ [cf. Eq. (1)] we find

$$d\ln(\delta\rho/\rho) = d\ln(\delta\sigma_{\rm osc})\,, \quad (42)$$

where we used Eq. (3) and considered that $\sigma_{\rm osc}$ does not have a scale dependence. In view of Eq. (9), Eqs. (41)

[2] Note that, in this case, the bound in Eq. (34) is not applicable because $H_{\rm dec} \sim \Gamma_\sigma < H_*$.

[3] We thank C. Gordon for reminding us of this issue.

and (42) show that *the contribution of the running of the amplification factor* ε^{-1} *to the spectral index* is

$$n_s - 1 = \frac{d \ln \varepsilon(N)}{dN}\,, \tag{43}$$

where N is the number of remaining e-foldings of inflation and we ignored the other contributions coming from the variation of H and also from the curvature of the curvaton potential.

Using Eqs. (5) and (6) we see that $\varepsilon = v_*/v_0$. Hence, since the observational bound on the spectral index is $|n_s - 1| \leq 10^{-2}$, we find the constraint

$$\frac{d \ln v(N)}{dN} \leq 10^{-2} \quad \Rightarrow \quad |(\dot{\phi}/\phi)_*| \ll H_*\,, \tag{44}$$

where we used that v is determined by the value of the radial field $v(t) = \phi(t)$ and that v_0 is a constant. From the above we realise that, in order not to violate the observational constraints regarding the scale invariance of the density perturbation spectrum, *the roll of the radial field has to be at most very slow when the cosmological scales exit the horizon*. However, the slow roll of ϕ cannot remain so indefinitely because we need $v_0 \gg v_*$ to have substantial amplification of the perturbations (i.e. $\varepsilon \ll 1$) and this has to be achieved in the remaining $N_* \lesssim 60$ e-foldings of inflation. Only then can the lower bound on the inflationary scale be substantially relaxed. Consequently, ϕ has to increase dramatically at some point *after* the exit of the cosmological scales from the horizon. This requirement is crucial for model-building.

B. Concrete examples

In this section we examine three specific scenarios for attaining low scale inflation. In each scenario we study the evolution of the radial field during inflation, estimate ε and determine the lowest allowed inflationary scale, which, according to Eq. (22), is given by

$$H_{\rm min} \sim \varepsilon^{4/5} \times 10^7 \text{ GeV}\,. \tag{45}$$

There are two ways of obtaining $\varepsilon < 1$. One way is to identify v with the minimum of the potential and assume that ϕ has reached it before the cosmological scales exit the horizon during inflation. This works only if $V(\phi)$ changes with time and, in particular, after the cosmological scales exit the horizon. Such a shift of the minimum of $V(\phi)$ is realisable if, for example, the radial field is coupled to some degree of freedom that changes value during inflation (e.g. the inflaton field itself). Another way of obtaining $\varepsilon < 1$ is to identify $v(t) = \phi(t)$ and consider that $\phi(t)$ is evolving during inflation, rolling toward its minimum but not having reached it yet, when the cosmological scales exit the horizon. In the following examples we employ both methods.

1. *Symmetry breaking during inflation*

We consider first the case that ϕ is initially held at the origin by an interaction with the inflaton field, being destabilised only when the inflaton field passes through some critical value[4]. This situation differs from the hybrid inflation mechanism only in that the would-be waterfall field ϕ is not responsible for the bulk of the inflationary potential, but instead is responsible for only a small fraction of it. The situation was actually described first a few years before hybrid inflation [17], the motivation then being the possibility of creating cosmic strings on cosmological scales (for a related situation see also Ref. [18]). To avoid misunderstanding, we emphasise that we allow inflation to be either hybrid or non-hybrid, it makes no difference to our considerations.

Due to the constraint from the spectral index we need the roll of the radial field to be slow at first, so that Eq. (44) is satisfied when the cosmological scales exit the horizon. However, we need the roll to accelerate later on so that the final value of ϕ can be much larger than ϕ_*. Therefore, we consider an effective running mass model for the radial field with $V(\phi) \simeq -\frac{1}{2} m_\phi^2(\phi)\phi^2$. The effective tachyonic mass $m_\phi(\phi)$ is such that, when the cosmological scales exit the horizon $m_\phi(\phi_*) \ll H_*$ and ϕ undergoes slow roll. However, at later times and near the VEV of ϕ we require $m_\phi(v_0) \gg H_*$ so that the total roll of ϕ after the exit of the cosmological scales is substantial.

To study this scenario we introduce the following toy-model:

$$V(\phi) = V_0 - \frac{1}{4}\lambda\phi^4 + \frac{\phi^{n+4}}{m_P^n}\,, \tag{46}$$

where $n \geq 1$. The above choice of model is somewhat contrived. Firstly, in a supersymmetric context a negative quartic term in the scalar potential can be generated only with the aid of additional fields (e.g. see [19]). Moreover, the quadratic term seems to be absent, which implies that supergravity corrections of order $\sim \pm H^2\phi^2$ to this potential must be suppressed and also the soft mass of the radial field must satisfy the bound

$$m_\phi < \sqrt{\lambda} H_*\,, \tag{47}$$

so that, since $\phi_* > H_*$ [5], the soft mass term contribution is negligible to the potential, when the cosmological scales exit the horizon. Note, however, that, if the radial field is kept originally at the origin due to an interaction

[4] Another possibility is that ϕ is locked on top of its potential hill by being coupled to an oscillating scalar field during inflation [15]. Further, the local maximum can be a point of enhanced symmetry for the radial field, which may be originally trapped inside it [16].

[5] The field is considered to be originally displaced from the top of the potential hill by its quantum fluctuations.

with the inflaton, then, immediately after the phase transition which releases the radial field, the soft mass term is almost canceled by the interaction term.

Despite the above peculiarities, let us see how such a model performs. The VEV of ϕ is

$$v_0 \sim \lambda^{1/n} m_P \ , \tag{48}$$

which means that

$$V_0 \sim \lambda^{(n+4)/n} m_P^4 \ . \tag{49}$$

The effective tachyonic mass of the rolling ϕ is

$$m_\phi^2(\phi < v_0) \simeq 3\lambda\phi^2. \tag{50}$$

The mass of ϕ in the vacuum is

$$\tilde{m}_\phi \sim \lambda^{(n+2)/2n} m_P \ . \tag{51}$$

Hence, for the above model to work we need that $\phi_* \leq \phi_c$, where

$$\phi_c \sim H/\sqrt{\lambda} \ , \tag{52}$$

is the critical value of the radial field where the field ceases to be light, i.e. $m_\phi(\phi_c) \sim H$. Obviously we also need $\phi_c < v_0$. The field must reach ϕ_c before the onset of the curvaton oscillations, but not necessarily during inflation. In all cases, though, it is easy to show that the slow roll of the field results in

$$\phi_* \sim \phi_c \ , \tag{53}$$

that is, the field is almost frozen during slow roll. In contrast, after ϕ_c has been reached the field rushes to its VEV in less than an e-folding (or equivalently a Hubble time).

Now, in order to avoid inflating the Universe due to the radial field, we require

$$\rho_c > V_0 \quad \Rightarrow \quad \phi_c > \sqrt{v_0 m_P} \sim \lambda^{2/n} m_P \ , \tag{54}$$

where ρ_c is the density of the Universe when the radial field reaches the value ϕ_c and we used that $V(\phi \ll v_0) \sim V_0$.

Let us now estimate the amplification factor for the curvaton's perturbations. The value of ε is determined by the radial field and is estimated as

$$\varepsilon = \frac{\phi_*}{v_0} \sim \frac{\phi_c}{v_0} \, . \tag{55}$$

In view of Eqs. (48) and (54) we find that the minimum accessible value is

$$\varepsilon_{\rm min} \sim \lambda^{1/n}. \tag{56}$$

Note that, since $\phi_c \geq H_*$, we always have $\varepsilon_{\rm min} > \varepsilon_{\rm MIN}$ [cf. Eq. (37)].

From the above, we see that a low inflationary scale can be accommodated only with very small values of λ. For example, inflation with $H_* \sim 1$ TeV can be attained only with $\lambda < 10^{-5n}$. As already discussed when this toy model was introduced, this is only one tuning problem of many.

For soft mass of order 1 TeV the above, in view of Eq. (47), implies immediately that $H_* > 1$ TeV always. In fact, using $m_\phi \sim 10^3$ GeV, Eqs. (45), (47) and (56) suggest that

$$H_{\rm min} \sim 10^{\frac{35n+24}{5n+8}} \ {\rm GeV} \, . \tag{57}$$

For $n = 2$ this suggests $H_{\rm min} \sim 10^5$ GeV with $\lambda \sim 10^{-4}$, whereas for $n = 4$ we readily obtain $H_{\rm min} \sim 10^6$ GeV with $\lambda \sim 10^{-6}$. Note that, for very large n (which, from Eq. (48), is equivalent to $v_0 \sim m_P$), we regain $H_{\rm min} \sim 10^7$ GeV.

Hence, even though somewhat contrived, this toy model shows that the PNGB mechanism for amplification of the curvaton's perturbations can indeed in principle work in the sense that the lower bound on the inflationary scale may be relaxed. Of course, in the symmetry breaking case considered here, it seems that the radial field must be a flat direction, protected by supergravity corrections. One then wonders why is it that the radial field itself is not considered as a curvaton candidate, as in Ref. [20].

An interesting variant of the above is considering a model based on the non-supersymmetric Coleman–Weinberg potential [21]:

$$V(\phi) = \frac{1}{4}\lambda\phi^4\left(\ln\frac{|\phi|}{v_0} - \frac{1}{4}\right) + \frac{1}{16}\lambda v_0^4, \tag{58}$$

which, for $\phi \ll v_0$ corresponds to a negative quartic potential (this could be thought as the $n = 0$ case of the above).

This time we find

$$V_0 \equiv V(\phi = 0) = \frac{1}{16}\lambda v_0^4. \tag{59}$$

The effective mass of the rolling ϕ is

$$\begin{aligned} m_\phi^2(\phi \ll v_0) \ &\simeq \ -|3 + \ln(|\phi|/v_0)|\lambda\phi^2 \\ &\Rightarrow \ \tilde{m}_\phi^2 = 3\lambda v_0^2, \end{aligned} \tag{60}$$

where $\tilde{m}_\phi$ is the mass of ϕ in the vacuum.

From Eq. (60) it is evident that the critical value ϕ_c, corresponding to the end of slow roll of the radial field, is the same as given in Eq. (52). Then, we readily obtain the constraint

$$\rho_c > V_0 \quad \Rightarrow \quad \phi_c > v_0^2/m_P \ . \tag{61}$$

Note that the above are equivalent with Eqs. (49), (50), (51) and (54) with the substitution: $v_0 \to \lambda^{1/n} m_P$ [cf. Eq. (48)].

Working in the same manner as previously, we find that

$$\varepsilon \sim \frac{\phi_c}{v_0} \ \Rightarrow \ \varepsilon_{\rm min} \sim \frac{v_0}{m_P} \, , \tag{62}$$

which, when inserted into Eq. (45) gives

$$H_{\min} \sim (v_0/m_P)^{4/5} \times 10^7 \text{ GeV}. \tag{63}$$

Hence, a relatively small value of v_0 can substantially relax the lower bound on the inflationary scale. For example, considering that v_0 is the Peccei–Quinn scale $v_0 \sim 10^{12}$ GeV, one obtains $H_{\min} \sim 100$ GeV.

2. *Smooth curvaton*

We turn our attention now to a more realistic suggestion, which facilitates the variation of the radial field ϕ through a coupling to the inflaton field s. What we have in mind is a model similar to smooth hybrid inflation [22], where the F-term scalar potential is of the form

$$V_F(\phi) = \left(\mu^2 - \frac{\phi^4}{m_P^2}\right)^2 + \frac{\phi^6 s^2}{m_P^4}\,, \tag{64}$$

where μ is some suitable mass scale. In contrast to smooth hybrid inflation we will consider that the vacuum energy responsible for inflation is not due to the above contribution to the scalar potential but due to some other s-dependent potential V_s such that

$$V_* \simeq V_s(s_*) \gg V_F(\phi_*, s_*)\,. \tag{65}$$

The full scalar potential of the radial field must also take supergravity corrections into account. Considering supergravity corrections, we have

$$V(\phi) \simeq -m_{\text{eff}}^2\phi^2 + V_F(\phi)\,, \tag{66}$$

where

$$m_{\text{eff}} \sim \max\{H, m_\phi\}\,, \tag{67}$$

being m_ϕ is the soft mass of the radial field and where we have assumed a negative effective mass-squared because the opposite case would send ϕ to zero and render the angular field (the curvaton) unphysical.

Suppose, at first, that the contribution to the effective potential due to supergravity corrections is negligible. This is so only if

$$m_{\text{eff}} < \frac{\mu\phi}{m_P}\,. \tag{68}$$

In this case the scalar potential for our radial field ϕ is the one shown in Eq. (64). Assuming that ϕ during inflation attains the value v_F which minimises V_F, we have $\phi \sim v_F$, where

$$v_F \sim \begin{cases} \mu m_P/s\,, & s > \sqrt{\mu m_P} \\ v_0 \sim \sqrt{\mu m_P}\,, & s \leq \sqrt{\mu m_P} \end{cases}\,. \tag{69}$$

Therefore, provided $V_s(s)$ is such that the inflaton field s is decreasing with time during inflation and also if $s_* \gg \sqrt{\mu m_P}$, we could have a gradual increase of ϕ due to the roll of s, after the cosmological scales exit the horizon. If the inflaton slow rolls then it is easy to see that the radial field will slow roll as well, in accordance to the requirement in Eq. (44). However, this is a drawback of this scenario because it cancels one of the advantages of inflation model–building under the curvaton hypothesis; that one can have fast–roll inflation, which does not require a flat direction for the inflaton field [6, 23].

Thus, in this case we may achieve an amplification factor ε^{-1} for the curvature perturbations given by

$$\varepsilon = \frac{v_F(s_*)}{v_0} \sim \frac{\sqrt{\mu m_P}}{s_*} \ll 1\,. \tag{70}$$

The above implies that low–scale inflation may indeed be achieved with a suitable value of μ. The minimum allowed value of ε is determined by the requirement in Eq. (68), which, when considering that $\phi \sim v_F$ as given by Eq. (69), suggests

$$\varepsilon_{\min} \sim \frac{m_{\text{eff}}}{\mu}\sqrt{\frac{m_P}{\mu}}\,. \tag{71}$$

Inserting the above into Eq. (45) and considering that $m_{\text{eff}} \sim H$ one finds the condition

$$H_{\min}\mu^6 \sim (10^{10}\text{ GeV})^7\,. \tag{72}$$

Hence we see that $H_{\min}$ is very sensitive to the value of μ. Bounds on the allowed range for μ are obtained as follows.

Firstly one needs to satisfy the bound in Eq. (65). Assuming $\phi \sim v_F$ during inflation we immediately obtain that

$$V_F(\phi = v_F) \sim \begin{cases} \mu^4\,, & s > \sqrt{\mu m_P} \\ \mu^3 s^2/m_P\,, & s \leq \sqrt{\mu m_P} \end{cases}\,. \tag{73}$$

Hence, we see that, $(V_F)_* \sim \mu^4$, which implies the upper bound

$$\mu^4 < V_* \sim (H_* m_P)^2\,. \tag{74}$$

A lower bound on μ is obtained from Eq. (68), when one considers $\phi_* \sim v_F(s_*)$ for $s_* > \sqrt{\mu m_P}$. Thus, one obtains the bound

$$\mu^3 > m_{\text{eff}}^2 m_P\,. \tag{75}$$

Note that, in view of Eq. (71), satisfying the above bound guarantees that $\varepsilon_{\min} < 1$.

Eqs. (74) and (75) result in the following allowed range for μ:

$$10.67 \leq \log(\mu/\text{ GeV}) \leq 11.13\,. \tag{76}$$

Using Eq. (72), the corresponding range for $H_{\min}$ is

$$10^4\text{ GeV} \leq H_{\min} \leq 10^7\text{ GeV}\,. \tag{77}$$

The above result shows that relatively low-scale inflation can be achieved in this case with the lowest scale $H_{\rm min}$ corresponding to the highest value of μ in the range shown in Eq. (76). We also found that we need $\mu \approx 10^{11}$ GeV for this scenario to work, which suggests that the VEV of ϕ may be comparable to the scale of grand unification $v_0 \sim 10^{15}$ GeV [6].

Here we should point out that there is another constraint that needs to be satisfied. We have to ensure that the coupling between the radial field and the inflaton field does not destabilise the slow roll of the inflaton, which is necessary for the spectral index bound in Eq. (44) to be satisfied, since the variation of ϕ is determined by the roll of the inflaton. Therefore, we need to ascertain that the mixed term in the scalar potential does not contribute to the effective mass of the inflaton enough for slow roll to be disturbed, i.e.

$$\phi_*^3/m_P^2 < H_* \sim m_{\rm eff} \,. \tag{78}$$

Comparing the above with the requirement in Eq. (68) we see that it is possible to satisfy the above provided Eq. (75) holds. Then, using that $\phi_* \sim v_F(s_*) \sim \mu m_P/s_*$ we find that the slow roll of the inflaton is not disturbed provided $s > s_f$, where

$$s_f \sim \mu \left(\frac{m_P}{m_{\rm eff}}\right)^{1/3} . \tag{79}$$

Note that, by virtue of Eq. (75), $\sqrt{\mu m_P} < s_f < m_P$. Therefore, in the range $s_f < s \leq m_P$ the inflaton slowly rolls toward the origin resulting also in the slow roll of ϕ as required. When the inflaton reaches the value s_f slow roll inflation is terminated and both s and ϕ rush toward their VEVs (unless the slow roll is interrupted earlier due to the form of $V_s(s)$).

Let us now consider the case when the supergravity corrections are not negligible. This case corresponds to:

$$m_{\rm eff} \geq \frac{\mu\phi}{m_P} \,. \tag{80}$$

Then the scalar potential can be approximated as

$$V(\phi) \simeq V_0 - m_{\rm eff}^2\phi^2 + \frac{\phi^8}{m_P^4} + \frac{\phi^6 s^2}{m_P^4} \,, \tag{81}$$

where V_0 is introduced to avoid negative contributions to the energy density, when ϕ assumes its VEV. The minimum of the above potential is given by

$$v_{\rm eff} \sim \begin{cases} \sqrt{m_{\rm eff}/s}\; m_P \,, & s > (m_{\rm eff} m_P^2)^{1/3} \\ (m_{\rm eff}/m_P)^{1/3} m_P \,, & s \leq (m_{\rm eff} m_P^2)^{1/3} \end{cases} \tag{82}$$

This time the VEV of ϕ is reached only when H reduced below the soft mass so that

$$m_{\rm eff} \to m_\phi \;\Rightarrow\; v_0 \sim (m_\phi/m_P)^{1/3} m_P \,. \tag{83}$$

For $m_\phi \sim 1$ TeV we have $v_0 \sim 10^{13}$ GeV. It is easy to see that the effective mass of the radial field is given by $m_{\rm eff} \sim H$, which means that the field is driven to the minimum $\phi \to v_{\rm eff}$ in less than an e-folding.

From the above we see that, if $V_s(s)$ is such that the inflaton field s is decreasing with time during inflation and also if $s_* \gg (H_* m_P^2)^{1/3}$, then we could have a gradual increase of ϕ due to the roll of s after the cosmological scales exit the horizon. However, when $s < (H_* m_P^2)^{1/3}$ the increase of $\phi \sim v_{\rm eff}$ is halted. In fact, after the end of inflation $v_{\rm eff}$ starts to *decrease* because $v_{\rm eff} \propto m_{\rm eff}^{1/3} \sim H^{1/3}(t)$ until $H \sim m_\phi$ when ϕ assumes its VEV v_0. This stage of decrease of v is counter productive as it increases ε.

This time, considering that $\phi \sim v_{\rm eff}$, we find

$$\varepsilon = \frac{v_{\rm eff}(s_*)}{v_0} \sim \left(\frac{m_P}{m_\phi}\right)^{1/3} \sqrt{\frac{H_*}{s_*}} \,. \tag{84}$$

Since we have $s_* \leq m_P$ the above suggests

$$\varepsilon_{\rm min} \sim 10^5 \sqrt{H_*/m_P} \,, \tag{85}$$

where we assumed $m_\phi \sim 1$ TeV. Inserting this into Eq. (45) it is easy to find that

$$H_{\rm min} \sim 10^6 \text{ GeV} \,. \tag{86}$$

Therefore, in this case it is again possible to lower somewhat the scale of inflation although this reduction is not dramatic[7].

Let us discuss briefly a few considerations regarding this case. At first, it is interesting to obtain the value of $V(v_{\rm eff})$ and compare it with V_* as was done in Eq. (74) for the previous case. This time, setting $V(v_0)$ to zero, we find

$$V_0 \sim \left(\frac{m_\phi}{m_P}\right)^{2/3} (m_\phi m_P)^2 \sim (10^8 \text{ GeV})^4 \ll V_* \,. \tag{87}$$

From the above it is evident that the contribution of the radial field to the total energy density during inflation is negligible as required.

Regarding the slow roll of the inflaton, using that $\phi \sim v_{\rm eff}$ it is straightforward to show that $\phi_*^6/m_P^4 < m_{\rm eff}^2 \sim H_*^2$ during inflation and, therefore, the slow roll of the inflaton is not disturbed. However, when s reduces below $(m_{\rm eff} m_P^2)^{1/3}$ we have

[6] Using Eqs. (71) and (72) it can be easily shown that $\varepsilon_{\rm min} \sim 10^4 (H_{\rm min}/10^{10}\text{GeV})^{2/3}$, while from Eqs. (37), (69) and (76) one readily obtains that $\varepsilon_{\rm MIN} \sim 10^{-4}(H_{\rm min}/10^{10}$ GeV). Hence, $\varepsilon_{\rm min} \gg \varepsilon_{\rm MIN}$ always. Note also, that, in view of the above and Eqs. (27) and (38) we find $\varepsilon'_{\rm MIN} \simeq \Omega_{\rm dec}\varepsilon_{\rm MIN} \leq \varepsilon_{\rm MIN}$.

[7] From Eqs. (37) and (83) it is easy to see that $\varepsilon_{\rm MIN} \sim 10^5 (H_{\rm min}/m_P)$. Comparing this with Eq. (85) it is straightforward to show that $\varepsilon_{\rm min} \gg \varepsilon_{\rm MIN}$ always. Moreover, using Eqs. (27) and (38) we find $\varepsilon'_{\rm MIN} \simeq 10^{-1}\Omega_{\rm dec}\varepsilon_{\rm MIN} < \varepsilon_{\rm MIN}$.

$v_{\rm eff}^6/m_P^4 \sim m_{\rm eff}^2 \sim H_*^2$ and the slow roll of both the inflaton and our radial field is terminated. Thus, this time

$$s_f \sim (m_{\rm eff} m_P^2)^{1/3}. \quad (88)$$

Finally, lets discuss also the value of μ in this case. μ has to satisfy the constraint in Eq. (80). Using Eq. (82) and taking $\phi \sim v_{\rm eff}$ it is easy to show that this constraint is always satisfied if $\mu < (H_{\rm min}^2 m_P)^{1/3} \sim 10^{10}$ GeV. For a given $H_* \geq H_{\rm min}$ one can find a lower bound on μ, above which the supergravity corrections are always subdominant. Indeed, considering also that $s_* \leq m_P$, Eqs. (80) and (82) suggest that the supergravity corrections cannot dominate for $\mu > \sqrt{H_* m_P}$. Since this would also imply that $V(\phi)$ would dominate V_* even if the supergravity corrections are negligible, we see that $\mu > \sqrt{H_* m_P}$ is excluded. The intermediate range of values for μ may allow a transition between the two cases during inflation.

To sum up, in the case of the "smooth curvaton" we have seen that, for rather natural values of the parameters, a moderate decrease of the inflationary energy scale is indeed possible. However, the model suffers from one disadvantage; namely that inflation has to be of the slow–roll type.

3. *The curvaton and the waterfall field*

As a final example we consider hybrid inflation [24], which, apart from the inflaton field s, introduces another so–called "waterfall" field Φ, which is responsible for the inflationary vacuum density. During inflation Φ is kept at the origin due to its interaction with the inflaton. However, after the inflaton decreases below a critical value, Φ is destabilised; it leaves the origin and rolls rapidly to its VEV M. At this point inflation is terminated.

In our example we consider a *negative* coupling between our radial field and the waterfall field of the sort that appears in models of inverted hybrid inflation [19]. This means that we will use the following scalar potential:

$$V(\phi) = V_0 - m_{\rm eff}^2\phi^2 - g^2\Phi^2\phi^2 + \frac{\phi^{n+4}}{m_P^n}\,, \quad (89)$$

where $n \geq 0$. The minimum of the above potential is located at

$$v = \left[(m_{\rm eff}^2 + g^2\Phi^2)m_P^n\right]^{1/(n+2)}. \quad (90)$$

Since the effective mass of the radial field is $m_\phi(\Phi) \sim \sqrt{m_{\rm eff}^2 + g^2\Phi^2} \geq m_{\rm eff} \geq H$ [cf. Eq. (67)] we expect ϕ to roll toward v in less than an e-folding.

During inflation $\Phi = 0$, while after inflation $\Phi = M$. Therefore, using Eq. (90) it is straightforward to show that

$$\varepsilon = \frac{v_*}{v_0} \sim \left(\frac{m_{\rm eff}}{gM}\right)^{2/(n+2)}, \quad (91)$$

where we assumed

$$g > m_{\rm eff}/M\,, \quad (92)$$

so that $\varepsilon < 1$. Using that $\varepsilon_{\rm MIN} \sim H_{\rm min}/v_0$ [cf. Eq. (37)], then Eqs. (90) and (91) give

$$\frac{\varepsilon}{\varepsilon_{\rm MIN}} \sim \left[\left(\frac{m_{\rm eff}}{H_{\rm min}}\right)^2\left(\frac{m_P}{H_{\rm min}}\right)^n\right]^{1/(n+2)}. \quad (93)$$

Typically in hybrid inflation the inflationary vacuum density is determined by the VEV of the waterfall field Φ. Therefore, we will consider

$$V_* \sim \alpha M^4 \quad \Rightarrow \quad H_* \sim \sqrt{\alpha}\, M^2/m_P\,, \quad (94)$$

where $\alpha \leq 1$ is a numerical coefficient. In certain supersymmetric realisations of hybrid inflation [25] α is expected to be close but smaller than unity, i.e. $\alpha \lesssim 1$. However, in supernatural [5] or running–mass hybrid inflation [26] the VEV of the waterfall field is typically $M \sim m_P$ with $V_* \sim 10^{10.5}$ GeV, which suggests that α is extremely small; $\alpha \sim 10^{-30}$.

Inserting Eqs. (91) and (94) into Eq. (22) we obtain

$$\left(\frac{M}{m_P}\right)^2 > \frac{10^{-11}}{\sqrt{\alpha}}\left(\frac{m_{\rm eff}}{gM}\right)^{8/5(n+2)}. \quad (95)$$

Now, during inflation it is easy to show that $V(v) \simeq V_0$, where

$$V_0 \sim \tilde{m}_\phi^2 v_0^2 \sim g^4 M^4 \left(\frac{m_P}{gM}\right)^{2n/(n+2)}, \quad (96)$$

with $\tilde{m}_\phi = m_\phi(\Phi = M)$ being the mass of ϕ in the vacuum. To ensure that the density of the radial field is subdominant during inflation we, therefore, require $V_* > V_0$. Employing Eqs. (94) and (96) this requirement becomes:

$$g^{n+4} < (\sqrt{\alpha})^{n+2}\left(\frac{M}{m_P}\right)^n. \quad (97)$$

Incorporating the above into Eq. (95) and using Eq. (94) we find:

$$\frac{H_*}{m_P} > \left[10^{-55}\left(\frac{m_{\rm eff}}{m_P}\right)^{\frac{8}{n+2}}\right]^{\frac{n+4}{5n+28}}. \quad (98)$$

Remarkably, Eq. (98) shows that *the lower bound on the inflationary scale is independent of α, that is of the VEV of the waterfall field*. In view of Eq. (67), we obtain

$$H_{\rm min} \sim 10^7 \text{ GeV} \times \begin{cases} 10^{-\frac{176}{(5n+6)(n+4)+4n}}\,, & n > 1 \\ \\ 10^{-\frac{16(2n+19)}{(5n+28)(n+2)}}\,, & n \leq 1 \end{cases}, \quad (99)$$

where we have used that $H_{\rm min} \leq m_\phi \sim 10^3$ GeV if $n \leq \sqrt{52} - 6 \simeq 1$. From the above we see that, for $n = 0$, $H_{\rm min}$ can be as low as $H_{\rm min} \sim 10$ GeV. Therefore, low scale inflation is indeed attainable. However, the lower bound on $H_{\rm min}$ increases with n as can be seen in Fig. 1.

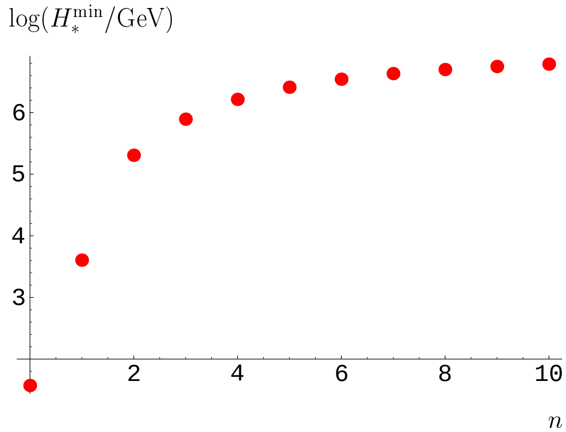

FIG. 1: Plot of $\log(H_*^{\rm min}/\ {\rm GeV})$ with respect to n in the case of a PNGB curvaton whose radial field is coupled to the waterfall field of hybrid inflation. It is evident that the larger the n is the tighter the lower bound on the inflationary scale becomes.

In view of the above results, it can be readily checked from Eq. (93) that $\varepsilon \gg \varepsilon_{\rm MIN}$ always[8].

There is one more condition which we need to verify. Comparing Eqs. (92) and (97) we find the condition:

$$\left(\frac{m_{\rm eff}}{M}\right)^{n+4} < (\sqrt{\alpha})^{n+2}\left(\frac{M}{m_P}\right)^n . \qquad (100)$$

For $H_* > m_\phi$ we have $m_{\rm eff} \sim H_*$ and, in view of Eq. (94), the above is equivalent to $H_* < m_P$, which is readily satisfied. For $H_* \leq m_\phi$, however, one needs to check the above condition for a given soft mass m_ϕ. Using $m_\phi \sim 10^3$ GeV we find that the above condition is equivalent to

$$H_* > 10^{-15(\frac{n+4}{n+2})} m_P . \qquad (101)$$

It can be checked that the above lower bound is less stringent than the ones appearing in Eq. (99) and, therefore, Eq. (100) is always satisfied, which means that there is always some available parameter space for g. Indeed, from Eqs. (92) and (97) is is easy to find the range of g for a given H_*:

$$\left(\frac{m_{\rm eff}}{H_*}\right)^2 \frac{H_*}{m_P} < \frac{g^2}{\sqrt{\alpha}} < \left(\frac{H_*}{m_P}\right)^{n/(n+4)} , \qquad (102)$$

[8] One can also show that $\varepsilon > \varepsilon'_{\rm MIN}$ for all n.

with the upper bound corresponding to $H_{\rm min}$. For example, for $H_* \sim 1$ TeV and $\alpha \sim 1$ the allowed range of g is $10^{-8} < g < 0.01 - 1$. Note, however, that, for $\alpha \ll 1$, g is bounded from above as $g < \alpha^{1/4}$. Indeed, for $H_* \sim 1$ TeV and $M \sim m_P$ we have $g \lesssim 10^{-8}$ for $n = 0$. Since, in this case, the radial field is probably a modulus, such a weak coupling is expected.

Due to the interaction between the radial field and the waterfall field it is possible that the mass of the PNGB curvaton can suddenly increase at the end of inflation. This may result in $f < 1$ and one may wonder whether further relaxation of the lower bound on the inflationary scale is possible. However, as we have already discussed [cf. Eq. (30)], the bound on H_* cannot be relaxed, even with $f \ll 1$, if $\delta \sim 1$ as is clearly the case here.

To summarise, we have shown that coupling the radial field to the waterfall field of hybrid inflation may allow low scale inflation with $H_* \sim 10$ GeV at best. This is quite a satisfactory result and corresponds to a rather realistic scenario, since hybrid inflation is well motivated by particle theory. Moreover, in contrast to the "smooth curvaton" example, we do not require slow roll inflation because, in this example, *the radial field does not roll at all during inflation*. Hence, one may have fast–roll hybrid inflation as discussed in Ref. [27]. The only potential difficulty is arranging for the coupling to be negative (see, however, Ref. [19]).

IV. THE CASE OF A HEAVY CURVATON

In this section we are going to considered the so called "heavy curvaton scenario" where an increment on the curvaton mass, at some moment after the end of inflation but before the onset of the curvaton oscillations, leads to a huge decrease of the inflationary scale through the attainment of a very small parameter δ [cf. Eq. (30)]. We will do so by the implementation of a second inflationary period following the idea first presented in [8]. We identify this second inflationary period as the thermal inflation one which triggers the increment on the curvaton mass when the flaton field, that responsible for the generation of the thermal inflation era, rolls down toward the minimum of the potential.

A. The thermal inflation model

Thermal inflation was introduced as a very nice mechanism to get rid of some unwanted relics that the main inflationary epoch is not able to dilute, without affecting the density perturbations generated during ordinary inflation. As its name suggests, thermal inflation relies on the finite-temperature effects on the "flaton" scalar potential. A flaton field could be defined as a field with a mass of the order 10^3 GeV, coming from the soft su-

persymmetric contributions[9], and a vacuum expectation value M much bigger than 10^3 GeV [28, 29]. After the period of reheating following the main inflationary epoch, the thermal background modifies the flaton potential trapping the flaton field at the origin and preventing it to roll-down toward M [30, 31]. At this stage

$$\begin{aligned} \rho = & \; V + \rho_T, \\ P = & \; -V + \tfrac{1}{3}\rho_T, \end{aligned} \tag{103}$$

making the condition for thermal inflation, $\rho + 3P < 0$, valid when the thermal energy density ρ_T falls below V_0, which corresponds to a temperature of roughly $V_0^{1/4}$. Thermal inflation ends when the finite temperature becomes ineffective at a temperature of order m_χ, so the number of e-folds is

$$\begin{aligned} N \;=\; & \ln\left(\frac{a_{\rm end}}{a_{\rm start}}\right) = \ln\left(\frac{T_{\rm start}}{T_{\rm end}}\right) \sim \\ \sim \; & \ln\left(\frac{V_0^{1/4}}{m_\chi}\right) \sim \frac{1}{2}\ln\left(\frac{M}{m_\chi}\right). \end{aligned} \tag{104}$$

Here we have used the fact that, in a flaton potential of the form

$$V = V_0 - (m_\chi^2 - gT^2) \mid \chi \mid^2 + \sum_{n=1}^{\infty} \lambda_n m_P^{-2n} \mid \chi \mid^{2n+4}, \tag{105}$$

where the nth term dominates:

$$\begin{aligned} \tilde{m}_\chi^2 \;=\; & 2(n+1)m_\chi^2, & (106) \\ M^{2n+2}m_P^{-2n} \;=\; & [2(n+1)(n+2)\lambda_n]^{-1}\tilde{m}_\chi^2, & (107) \\ V_0 \;=\; & [2(n+2)]^{-1}\tilde{m}_\chi^2 M^2. & (108) \end{aligned}$$

It is worthwhile to mention that the potential is stabilized by non-renormalisable terms. Otherwise, the vacuum expectation value M would not be much bigger than $\tilde{m}_\chi$, spoiling the suppression of the unwanted relics.

Guided by the result in [8] we proceed to implement a second inflationary stage into our curvaton scenario in order to lower the main inflationary energy scale. If this second epoch of inflation is the thermal inflation one devised in [28, 29, 30] we would be solving not only the issue of the ordinary inflation energy scale but also the moduli problem present in the standard cosmology.

In this new scenario the scalar potential would be composed of the usual potential terms for the curvaton and the flaton fields, as well as for a quartic interaction term between the two fields:

$$\begin{aligned} V(\chi,\sigma) \;=\; & V_0 - (m_\chi^2 - gT^2)|\chi|^2 + m_\sigma^2|\sigma|^2 + \lambda|\chi|^2|\sigma|^2 + \\ + & \sum_{n=1}^{\infty}\lambda_n m_P^{-2n}|\chi|^{2n+4}. \end{aligned} \tag{109}$$

[9] As an example we are going to focus in a value for the gravitino mass $m_{3/2}$ of order $m_{3/2} \sim 10^3$ GeV which comes from gravity-mediated SUSY breaking.

The λ term is the one which will increment the mass of the curvaton field when the flaton acquires its vacuum expectation value at the end of thermal inflation.

Let's assume that the usual inflation and its corresponding reheating have already happened, so that the flaton and the curvaton fields are embedded into a radiation bath. Therefore, even when the minimum of the potential is located at $\chi = M_\chi(\sigma_*) \neq 0$ and $\sigma = 0$, χ is trapped at the origin because of the finite-temperature effects and $\mid \sigma \mid = \sigma_* \neq 0$ because $m_\sigma < H < H_*$. Thus, the value of the scalar potential at this stage is:

$$V(\chi = 0, \sigma = \sigma_*) = V_0 + m_\sigma^2\sigma_*^2, \tag{110}$$

with

$$\begin{aligned} \tilde{m}_\chi^2 \;=\; & 2(n+1)(m_\chi^2 - \lambda \mid \sigma \mid^2), & (111) \\ M_\chi^{2n+2}m_P^{-2n} \;=\; & [(n+2)\lambda_n]^{-1}(m_\chi^2 - \lambda \mid \sigma \mid^2), & (112) \\ V_0 \;=\; & [2(n+2)]^{-1}(\tilde{m}_\chi^2 M_\chi^2) \mid_{\sigma=0}. & (113) \end{aligned}$$

When the thermal energy density falls below $V_0 + m_\sigma^2\sigma_*^2$ thermal inflation begins. This period lasts until the temperature is of the order the effective mass of the flaton field which is $\tilde{m}_\chi = (m_\chi^2 - \lambda\sigma_*^2)^{1/2}$. Note that $\lambda\sigma_*^2 < m_\chi^2$ because otherwise there is no thermal inflation. Then, we obtain a first constraint on the value of the parameter λ:

$$\lambda < \frac{m_\chi^2}{\sigma_*^2} \sim \frac{10^{-2}\ \mathrm{GeV}^2}{H_*^2\Omega_{\rm dec}^2}, \tag{114}$$

where we have used the Eq. (11) and focused on $m_\chi \sim 10^3$ GeV which comes from the gravity-mediated SUSY breaking contributions.

When thermal inflation ends the thermal energy density is no longer dominant. The Hubble parameter at the end of thermal inflation is then associated to the energy density coming from the curvaton and the flaton fields:

$$H_{\rm osc}^2 = \frac{\rho_T + V(\chi = 0, \sigma = \sigma_*)}{3m_P^2} \sim \frac{m_\chi^2 M^2}{3m_P^2}, \tag{115}$$

so that

$$H_{\rm osc} \sim 10^{-16}M, \tag{116}$$

and therefore the parameter f [cf. Eq. (13)] is

$$f \equiv \frac{H_{\rm osc}}{\tilde{m}_\sigma} \sim 10^{-16}\frac{M}{\tilde{m}_\sigma}, \tag{117}$$

where $M \equiv M_\chi \mid_{\sigma=0}$ is somewhere in the range $10^3\ \mathrm{GeV} \ll M \leq 10^{18}$ GeV.

With this so-low value for the Hubble parameter at the end of thermal inflation, the parameter δ [cf. Eq. (27)] is

$$\delta \sim 10^{-8}\sqrt{\frac{M}{H_*}}, \tag{118}$$

so that the bounds in Eq. (22) and Eq. (26) become[10]:

$$H_* > 10^{-6}\ \mathrm{GeV}\ \frac{M^{4/5}}{\tilde{m}_\sigma^{4/5}\Omega_{\mathrm{dec}}^{2/5}}, \tag{119}$$

$$H_* > 10^{-7}\ \mathrm{GeV}^{1/2}\ \frac{M^{3/4}}{\tilde{m}_\sigma^{1/4}\Omega_{\mathrm{dec}}^{1/2}}. \tag{120}$$

The effective mass of the curvaton field after the end of thermal inflation, i.e., when $\bar{\chi} = M_\chi$ and $\bar{\sigma} = 0$ are the average values of the flaton and the curvaton fields, is

$$\tilde{m}_\sigma = (m_\sigma^2 + \lambda M^2)^{1/2}. \tag{121}$$

Note that we are focusing in the case which refers to a final curvaton decay rate Γ_σ smaller than the Hubble parameter at the beginning of the oscillations H_{osc}. This is to make the curvaton field decay before the flaton field so that we can keep working in the simplest curvaton scenario where the curvaton field oscillates in a radiation background [2, 10].

Making use of the constraint in Eq. (114) and the expression in Eq. (121), and taking into account that the bare curvaton mass m_σ is smaller than the Hubble parameter H_{osc} at the end of thermal inflation, we obtain an upper bound on the effective mass of the curvaton field:

$$\tilde{m}_\sigma < 10^{-1}\ \mathrm{GeV}\frac{M}{H_*\Omega_{\mathrm{dec}}}. \tag{122}$$

This bound is consistent with Eq. (119). When the Eq. (122) is applied to the Eq. (120), we obtain a lower bound for H_* which is consistent with low-energy scale inflation:

$$H_* > 10^{-9}\ \mathrm{GeV}^{1/3}M^{2/3}. \tag{123}$$

The last inequality is stronger than that of Eq. (119) only while the effective mass of the curvaton field is

$$\tilde{m}_\sigma > 10^2\ \mathrm{GeV}^{10/11}M^{1/11}\Omega_{\mathrm{dec}}^{2/11}; \tag{124}$$

otherwise, we still need to consider the expression in Eq. (119).

B. Required parameter space

Once we have checked the viability of a low-energy scale inflation we proceed to investigate the required range of values for the parameters of the Lagrangian. Hereafter we are going to focus on the gravity-mediated SUSY breaking scheme where the Hubble parameter during inflation is $H_* \sim m_{3/2} \sim 10^3$ GeV. After thermal inflation has ended, the flaton and curvaton fields start to oscillate, eventually decaying into thermalised radiation. The decay process is distinguished by the decay rate. The field with the biggest decay rate will decay first. The flaton and curvaton decay rates are given by

$$\Gamma_\chi \approx \gamma_\chi \frac{m_\chi^3}{M^2} \quad \text{and} \quad \Gamma_\sigma \approx \gamma_\sigma \frac{\tilde{m}_\sigma^3}{m_P^2}, \tag{125}$$

with $\gamma_\chi \lesssim 1$ and $\gamma_\sigma \gtrsim 1$. Since the curvaton mechanism must not be modified, the flaton field must decay well before the curvaton decay. This requires

$$\tilde{m}_\sigma^3 \ll m_\chi^3 \frac{m_P^2}{M^2} \sim \frac{10^{46}\ \mathrm{GeV}^5}{M^2}. \tag{126}$$

Now, using the expression in Eq. (119), which is valid for $\tilde{m}_\sigma \leq 10^2\ \mathrm{GeV}^{10/11}M^{1/11}\Omega_{\mathrm{dec}}^{2/11}$ [cf. Eq. (124)], we require

$$\tilde{m}_\sigma > 10^{-11}M\Omega_{\mathrm{dec}}^{-1/2}, \tag{127}$$

in order to obtain low-energy scale inflation. Note that, combining the above with Eq. (117), we find

$$f < 10^{-5}\sqrt{\Omega_{\mathrm{dec}}} \ll 1, \tag{128}$$

as required by the heavy curvaton scenario. Similarly to the above, using the expression in Eq. (120), which is valid for $\tilde{m}_\sigma > 10^2\ \mathrm{GeV}^{10/11}M^{1/11}\Omega_{\mathrm{dec}}^{2/11}$ [cf. Eq. (124)], we require

$$\tilde{m}_\sigma > 10^{-40}\mathrm{GeV}^{-2}M^3\Omega_{\mathrm{dec}}^{-2}. \tag{129}$$

Thus, for values of $\tilde{m}_\sigma$ less than $10^2\ \mathrm{GeV}^{10/11}M^{1/11}\Omega_{\mathrm{dec}}^{2/11}$ the required range of values for $\tilde{m}_\sigma$ is:

$$10^{-11}M < \tilde{m}_\sigma < 10^2\ \mathrm{GeV}^{10/11}M^{1/11}, \tag{130}$$

where the lower bound comes from the Eq. (127). The vacuum expectation value M is in the range

$$10^{12}\ \mathrm{GeV} \lesssim M \lesssim 10^{14}\ \mathrm{GeV}. \tag{131}$$

where the lower bound comes from the solution to the moduli problem as we will see later, and the upper bound comes from the Eq. (130). On the other hand, for values of $\tilde{m}_\sigma$ bigger than $10^2\ \mathrm{GeV}^{10/11}M^{1/11}\Omega_{\mathrm{dec}}^{2/11}$ the required range of values for $\tilde{m}_\sigma$ is:

$$\max\{10^2\ \mathrm{GeV}^{10/11}M^{1/11}, 10^{-40}\ \mathrm{GeV}^{-2}M^3\} < \\ < \tilde{m}_\sigma < 10^{15}\ \mathrm{GeV}^{5/3}/M^{2/3}, \tag{132}$$

where we have used the Eqs. (126) and (129), and M can be up to m_P. We have considered all the other possible constraints on $\tilde{m}_\sigma$ and found they are irrelevant compared with those in Eq. (130) and Eq. (132).

[10] In the heavy curvaton mechanism $\varepsilon = 1$ because there is no amplification of the curvaton perturbations.

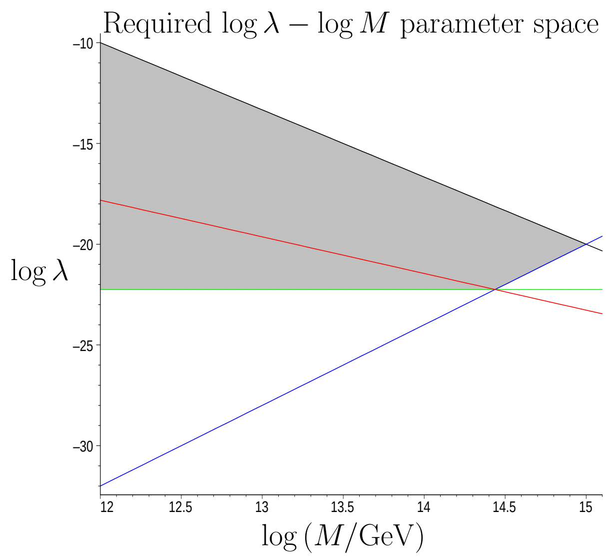

FIG. 2: Required $\lambda - M$ parameter space (grey region) as a logarithmic plot. The two lines in the middle of the graph correspond to the limits in Eq. (130) which are valid up to the meeting point of the three lowest lines. The slanting lines correspond to the limits in Eq. (132). Note that it is impossible to satisfy the conditions in Eq. (132) beyond the meeting point of the uppest and the lowest lines.

Figure 2 shows the required parameter space λ vs M (grey region) as a logarithmic plot. We have made use of the definition of the curvaton effective mass $\tilde{m}_\sigma$ in terms of the coupling constant λ and the vacuum expectation value M:

$$\tilde{m}_\sigma^2 \approx \lambda M^2, \tag{133}$$

and the required parameter space $\tilde{m}_\sigma$ vs M studied before. Note that for values of M higher than $\sim 10^{15}$ GeV it is impossible to satisfy the Eq. (132), so our final range for M is

$$10^{12}\ \mathrm{GeV} \lesssim M \lesssim 10^{15}\ \mathrm{GeV}. \tag{134}$$

The required values for λ are in agreement with the upper bound in the Eq. (114):

$$\lambda < \frac{10^2\ \mathrm{GeV}^2}{H_*^2} \sim 10^{-4}, \tag{135}$$

and with the lower bound

$$\lambda > \frac{H_{\rm osc}^2}{M^2} \approx \frac{m_\chi^2}{3m_P^2} \sim 10^{-31}, \tag{136}$$

which follows from $\tilde{m}_\sigma > H_{\rm osc}$.

Once we have found the required parameter space for λ we must do the same for the other relevant parameter of the Lagrangian: the bare mass of the curvaton m_σ. The first bound we have to take into account is

$$m_\sigma < H_{\rm osc} \sim 10^{-16} M, \tag{137}$$

which is related to the fact that the oscillations of the curvaton around the minimum begin due to the sudden increment in the curvaton mass at the end of thermal inflation. There are other two bounds on m_σ we should take into account. In order to get these bounds, and to understand the lower bound $M \gtrsim 10^{12}$ GeV, we must study the solution to the moduli problem.

C. Solution to the moduli problem

Among the unwanted relics that the inflationary epoch is not able to dilute are the moduli. Moduli fields are flaton fields with a vacuum expectation value of order the Planck mass. The decays of the flaton and the curvaton fields increment the entropy, so that the big-bang moduli abundance, defined as that produced before thermal inflation and given by [28]

$$\frac{n_\Phi}{s} \sim \frac{\Phi_0^2}{10 m_P^{3/2} m_\Phi^{1/2}}, \tag{138}$$

where Φ_0 is the vacuum expectation value of the moduli fields, gets suppressed by three factors:

$$\Delta_{PR} \simeq \frac{g_*(T_{PR})}{g_*(T_C)} \frac{T_{PR}^3}{T_C^3}, \tag{139}$$

due to the parametric resonance process following the end of the thermal inflation era, where T_{PR} is the temperature just after the period of preheating and T_C is the temperature at the end of thermal inflation,

$$\Delta_\chi \simeq \frac{4\beta V_0/3T_\chi}{(2\pi^2/45) g_*(T_{PR}) T_{PR}^3}, \tag{140}$$

due to the flaton decay, where T_χ is the temperature just after the decay[11], and β is the fraction of the total energy density left in the flatons by the parametric resonance process ($\beta \sim 1$), and

$$\Delta_\sigma \simeq \frac{4m_\sigma^2 \sigma_{\rm osc}^2/3\Omega_{\rm dec} T_{\rm dec}}{(2\pi^2/45) g_*(T_\chi) T_\chi^3}, \tag{141}$$

due to the curvaton decay, where $T_{\rm dec}$ is the associated reheating temperature which must be bigger than 1 MeV

[11] This is assuming that the flaton has come to dominate the energy density just before decaying.

not to disturb the nucleosynthesis process[12]. This enhancement in the entropy depends on the temperature just after the flaton decay

$$T_\chi \simeq \frac{10^{13}\ \mathrm{GeV}^2}{M}\gamma_\chi^{1/2}, \tag{142}$$

which is obtained by setting $\Gamma_\chi \sim H$ and assuming that the flaton decay products thermalise promptly. Thus, the abundance of the big-bang moduli after thermal inflation is:

$$\begin{aligned}\frac{n_\Phi}{s} &\sim \frac{\Phi_0^2}{10 m_P^{3/2} m_\Phi^{1/2}\Delta_{PR}\Delta_\chi\Delta_\sigma} \sim \frac{\Phi_0^2 T_\chi^4 T_{\rm dec} T_C^3}{10^5\beta V_0 m_\Phi^{1/2} m_\sigma^2 \Omega_{\rm dec} m_P^{3/2} H_*^2}\\ &\sim 10^{48}\ \mathrm{GeV}^8\, m_\sigma^{-2} M^{-6}\gamma_\chi^2 \left(\frac{\Phi_0}{m_P}\right)^2 \left(\frac{T_{\rm dec}}{1\ \mathrm{MeV}}\right)\left(\frac{T_C}{m_\Phi}\right)^3 \times\\ &\times \left(\frac{m_\Phi}{10^3\ \mathrm{GeV}}\right)^{1/2}\frac{1}{\beta}\left(\frac{m_\Phi^2 M^2}{V_0}\right)\frac{1}{\Omega_{\rm dec}}\left(\frac{10^3\ \mathrm{GeV}}{H_*}\right)^2. \end{aligned} \tag{143}$$

The lower bound

$$m_\sigma \gtrsim \frac{10^{30}\ \mathrm{GeV}^4}{M^3}, \tag{144}$$

is obtained when taking into account the restriction $n_\Phi/s \lesssim 10^{-12}$ coming from nucleosynthesis [32].

Let's have a look at the thermal inflation moduli abundance defined as that produced after thermal inflation:

$$\begin{aligned}\frac{n_\Phi}{s} &\sim \frac{\Phi_0^2 V_0^2/10 m_\Phi^3 m_P^4}{(2\pi^2/45)g_*(T_{PR})T_{PR}^3\Delta_\chi\Delta_\sigma} \sim \frac{\Phi_0^2 V_0 T_\chi^4 T_{\rm dec}}{10^9\beta m_\Phi^3 m_\sigma^2\Omega_{\rm dec} m_P^4 H_*^2}\\ &\sim 10^{-6}\ \mathrm{GeV}^4 m_\sigma^{-2} M^{-2}\gamma_\chi^2\left(\frac{\Phi_0}{m_P}\right)^2\left(\frac{T_{\rm dec}}{1\ \mathrm{MeV}}\right)\frac{1}{\beta}\times\\ &\times\left(\frac{10^3\ \mathrm{GeV}}{m_\Phi}\right)\left(\frac{V_0}{m_\Phi^2 M^2}\right)\frac{1}{\Omega_{\rm dec}}\left(\frac{10^3\ \mathrm{GeV}}{H_*}\right)^2. \end{aligned} \tag{145}$$

To suppress the thermal inflation moduli at the required level $n_\Phi/s \lesssim 10^{-12}$ we require

$$m_\sigma \gtrsim \frac{10^3\ \mathrm{GeV}^2}{M}. \tag{146}$$

The Eqs. (144) and (146) are the other two bounds we have talked about before, and we have to complement them with that already found in Eq. (137):

$$\max\left\{\frac{10^{30}\ \mathrm{GeV}^4}{M^3}, \frac{10^3\ \mathrm{GeV}^2}{M}\right\} \lesssim m_\sigma \lesssim 10^{-16} M \tag{147}$$

This required parameter space is shown in Figure 3 (grey region) as a logarithmic plot.

[12] We have assumed that ρ_χ does not change appreciably from the time when $T = T_C$ to the time when $T = T_\chi$. This is a good approximation since $\Gamma_\chi \gg \Gamma_\sigma$.

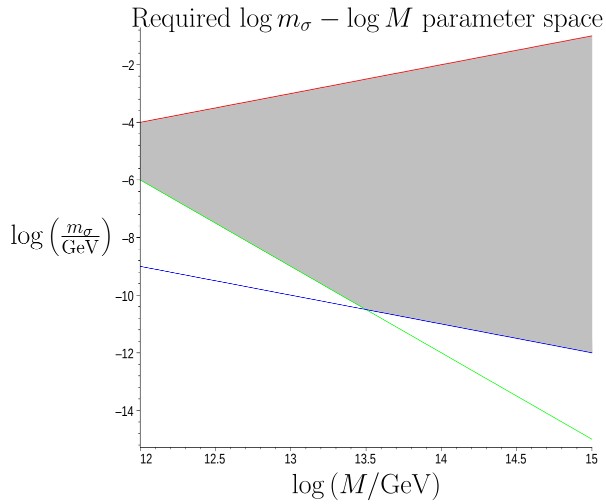

FIG. 3: Required $m_\sigma - M$ parameter space (grey region) from Eq. (147) as a logarithmic plot. The smallness of the bare curvaton mass m_σ suggests the curvaton could be a PNGB.

The Eqs. (143) and (145) give us information about the necessary conditions for the suppression of the big-bang and thermal inflation moduli, but they are based on the unknown parameters M and m_σ. Since we still need to know if the range $M \lesssim 10^{15}$ GeV, required to obtain a low-energy scale inflation, is not forbidden by the requirements coming from the solution to the moduli problem, we must find a m_σ-independent relation on M. This relation can be found noting that the increment in the entropy due to the curvaton decay (Eq. 141) can be written in an alternative way:

$$\Delta_\sigma \simeq \left[\frac{g_*(T_{\rm dec})}{g_*(T_\chi)(1-\Omega_{\rm dec})^3}\right]^{1/4}, \tag{148}$$

so the abundance of big-bang moduli after thermal inflation is:

$$\begin{aligned}\frac{n_\Phi}{s} &\sim \frac{\Phi_0^2}{10\, m_P^{3/2} m_\Phi^{1/2}\Delta_{PR}\Delta_\chi\Delta_\sigma} \sim \frac{10\Phi_0^2 T_\chi T_C^3(1-\Omega_{\rm dec})^{3/4}}{\beta V_0 m_\Phi^{1/2} m_P^{3/2}}\\ &\sim 10^{24}\mathrm{GeV}^3 M^{-3}\gamma_\chi^{1/2}(1-\Omega_{\rm dec})^{3/4}\left(\frac{\Phi_0}{m_P}\right)^2\times\\ &\times\left(\frac{T_C}{m_\Phi}\right)^3\left(\frac{m_\Phi}{10^3\ \mathrm{GeV}}\right)^{1/2}\frac{1}{\beta}\left(\frac{m_\Phi^2 M^2}{V_0}\right). \end{aligned} \tag{149}$$

This means that

$$M \gtrsim 10^{12}\ \mathrm{GeV}, \tag{150}$$

to satisfy $n_\Phi/s \lesssim 10^{-12}$. This is the lower bound on M we have used throughout the paper.

Of course we might have considered the scenario where there are no moduli fields at all. Without the introduction of the moduli problem the Eqs. (144) and (146)

become unnecessary, and the two lowest lines in Figure 3 disappear. This does not help for the improvement of the required range of values for m_σ but it does for λ as the lower bound on M in Eq. (150) becomes replaced by $M \gg 10^3$ GeV, which comes from the definition of the flaton fields. In this way the range of values for M extends to smaller values well below 10^{12} GeV until the coupling constant λ eventually reaches the lower bound 10^{-3}.

The introduction of a period of thermal inflation into our curvaton scenario has helped us not only to lower the energy scale of the main inflationary epoch, but also to solve the moduli problem still present after ordinary inflation. The required parameter space has been sketched in Fig. 2 and 3, and the vacuum expectation value for the flaton field has been showed to be in the range 10^{12} GeV $\lesssim M \lesssim 10^{15}$ GeV. The required parameter space $\tilde{m}_\sigma - M$ suggests the curvaton field could be a PNGB [14]. This is because in the presence of supergravity all the scalar fields, whose masses are not protected by a global symmetry, acquire soft masses of the order of the gravitino mass. The smallness of the curvaton mass is in turn because of the very small value for $H_{\rm osc}$. The parameter $H_{\rm osc}$ is directly proportional to M, so the bigger M is, the more possible to obtain a range of values for m_σ compatible with the soft supersymmetric contributions. A higher required value for M might be achieved in a scenario with two bouts of thermal inflation as suggested in [28]. This would open the possibility of obtaining a higher range of required values for m_σ, but more investigation is needed since the scenario proposed in [28] might change with the introduction of the curvaton field. We should also look for a mechanism to improve the required range of values for the coupling constant λ in presence of the moduli problem.

V. CONCLUSIONS

We have presented two different types of curvaton scenario, in which the Hubble scale of inflation can be much lower than $H_* \sim 10^7$GeV, which is the default lower bound for the standard curvaton model [7]. The first of these scenarios considers a PNGB curvaton, whose order parameter increases after the cosmological scales exit the horizon during inflation. The second scenario considers a curvaton, whose mass, being appropriately Higgsed, is substantially enlarged at a phase transition after the end of inflation ("heavy curvaton"). We have shown that both of these mechanisms are indeed able to accommodate inflation scales as low as $H_* \sim 1$ TeV or even lower.

In particular, in the case of a PNGB curvaton, we have derived that the lower bound on H_* is reduced as $\varepsilon^{4/5}$, where $\varepsilon \sim v_*/v_0$ is a measure of the growth of the order parameter v. We have shown that, the rate of variation of the order parameter (determined by the value of the radial field of our PNGB curvaton) during inflation, when the cosmological scales exit the horizon, should not be too large because, otherwise, it endangers the scale invariance of the density perturbation spectrum. Hence, the radial field must at most slow–roll when the cosmological scales exit the horizon. However, we need a substantial total variation of the radial field to achieve an adequately small ε. This turns up to be a tough requirement to meet in model-building. Nevertheless we did come up with a number of successful realisations. In fact, we studied three different models for attaining a small ε. The first model considers a symmetry breaking during inflation, which releases the radial field associated with our PNGB from the origin. To preserve scale invariance we have seen that a running tachyonic mass is required so that our radial field slow rolls at first but runs faster after the exit of the cosmological scales from the horizon. Our results showed that, by tunning the parameters, a moderate relaxation of the lower bound to the inflationary scale can indeed be achieved in the supersymmetric case, when the potential is stabilised by non-renormalisable terms. A renormalisable non-supersymmetric Coleman–Weinberg potential, though, does much better without any significant tunning. The second model assumes a suitable coupling between the inflaton field and the radial field, such that the order parameter of the PNGB is modulated by the variation of the inflaton. The model is similar to smooth hybrid inflation in the sense that the radial field, which lies at the temporary minimum of its potential, slowly moves away from the origin as its potential changes due to the roll of the inflaton. Again we find that a moderate relaxation of the lower bound on the inflationary scale is possible. However, one needs to constrain the vacuum expectation value (VEV) of the radial field accordingly, at some appropriate intermediate scale. Another disadvantage is that we need slow–roll inflation in order to avoid spoiling the scale invariance of the perturbation spectrum, even though these perturbations are not due to the inflaton field. Our third model is the most promising one. Here we introduced a coupling between the radial field of our PNGB curvaton and the waterfall field of hybrid inflation. The latter is kept at the origin during inflation, which means that our radial field (which again lies at the temporary minimum of its potential) also remains constant. At the end of inflation, however, the waterfall field rushes toward its VEV. Consequently the potential of our radial field changes accordingly and the radial field grows substantially, allowing for a really small ε. In this case, we have found that the we can achieve inflation with Hubble scale as low as $H_* \sim 10$ GeV regardless of the value of the VEV of the waterfall field and only with a reasonable upper bound on the coupling. Note also, that the model works well even when considering fast–roll hybrid inflation [27].

In the heavy curvaton scenario we have worked as follows. We have implemented the idea of a thermal inflation epoch, introduced in Refs. [28, 29, 30] to solve the moduli problem, as a second inflationary period necessary to lower the energy scale of the main inflationary stage. In our model, a flaton field χ

with bare mass coming from soft supersymmetric contributions and vacuum expectation value in the range 10^{12} GeV $\lesssim M \lesssim 10^{15}$ GeV, is held at the origin of the scalar potential by finite-temperature effects. These effects are associated to the thermal background created by the main reheating epoch. When temperature falls below V_0 thermal inflation begins. This period of thermal inflation lasts around ten e-folds until the temperature falls below m_χ liberating the flaton field to roll away toward the minimum of the potential. The curvaton field is coupled to the flaton one so its mass is largely increased at the end of thermal inflation. This increment is enough to lower the bound on H_* to satisfactory levels, without sending the non-gaussianity constraint to the limit. However, the energy scale of the thermal inflation epoch is very small, requiring in turn a bare mass for the curvaton field of at most $10^{-4} - 10^{-1}$ GeV. Taking into account the soft supersymmetric contributions to m_σ, the required smallness of m_σ points toward using a PNGB curvaton to achieve low-scale inflation.

Neither of the types of mechanism that we presented is completely compelling. The first one suffers from the problem of arranging for the appropriate behaviour of the radial field of the PNGB curvaton. The value of the radial field should undergo significant growth in total but, when the cosmological scales exit the horizon, the field should be at most slowly rolling. One possibility is considering a phase transition leading to spontaneous symmetry breaking. Just after the transition the effective tachyonic mass of the radial field is suppressed and the field is indeed slowly rolling. But why does the phase transition take place during inflation at a suitable time, instead of much earlier or later? That is a good question, though it has not prevented several authors from invoking such a phase transition for other purposes. It might perhaps be natural if the inflaton and the symmetry-breaking field are related in some way. For example, one possibility is considering inflation, which lasts only a limited number of e-foldings, such as fast–roll inflation [33] or locked inflation [15]. In this case, it is possible that the required phase transition occurs at (or just after) the onset of inflation [34]. Other solutions to the problem of the behaviour of the radial field require particular realisations, which may well work (as we showed) but the results are highly model dependent.

The second mechanism suffers from the problem that the mass of the curvaton before oscillation, as well as its coupling, have to be much smaller than one would expect. In a companion paper [13] it will be shown how this tuning problem can be at least alleviated.

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